JS Squared

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SUBJECT Deliverable D7: Engineering Models

Purpose: The purpose of this memorandum is to analyze a solution from one aspect to develop an engineering model, which helps to further understand the design.

Conclusions: An analysis that calculates the shear force and bending moment will help to determine if the ladder is suitable for the intended application. The ladder that was designed will be able to sufficiently sustain a user who is 300 lbs, which satisfies one of the requirements and is deemed suitable for use in that regard.

Process: The first step in the creation of the model was making assumptions to streamline the model. These are attached below and include things such as neglecting the weight of the ladder. After that, a simplified drawing was created to reinforce what is being analyzed in this problem. Once the simplified drawing was created, the free body diagrams were drawn to indicate the forces that are being analyzed. The summation of the forces in certain directions and moments about certain points yielded derived equations for the shear force and bending moment diagrams. Once the equations were derived, a MATLAB script was created to eventually create graphs and process the data analytically. Additionally, a screenshot from an online calculator is attached to verify the results obtained in the MATLAB script.

Discussion: The team had an expectation that the ladder should be able to comfortably support the weight of a 300 lb user. The model that was created analyzed the shear force and bending moment that a 300-pound individual who stood at the center of one of the rungs would exert. With regards to the shear force, it was relatively straightforward since the person's weight is considered as a concentrated load. From the left-hand side to the center of the rung, the result indicated a shear force of 668 Newtons, and from the center of the rung to the right-hand side, the result indicated a shear force of -668N. This means that if anyone were to virtually cut anywhere along the left section or the right section, a force that runs parallel to the vertical cut would yield 668 and -668N respectively. A similar thing can be said about the bending moment. Due to the nature of the supports at the end, it has a reaction moment, and so at the left-hand and right-hand edges, there is a -67.8 N-m bending moment. In addition, the bending moment goes up to a high point of 67.7 N-m at the point where the load is applied before coming back down to -67.8 N-m at the right-hand side. Overall, the model improved understanding by allowing the team to see how the loads would be applied on a real ladder such as this one. It is useful to see how the shear force and bending moment look like, since that gives an insight into stress, strain, and even factor of safety and understanding when such a material would fail. However, the model is limited because it doesn't account for two contact points for two legs that the average college student would have. It is also limited because it doesn't account for the weight of the ladder. For further iterations, the team should focus on reducing the weight of the ladder, since that would allow for more margin with respect to the user's weight when designing such a solution.

Sidh Gurnani

Attachments: Simplifying Assumptions, P-Diagram, Design Variables, Simplifying Model Sketch, Free Body Diagrams, Step-by-Step Derivation, Final Equations, Engineering Units Equation Validation Check, Picture of Code Developed, Model Results Graph (in MATLAB), Model Results Graph (from an online calculator)

Simplifying Assumptions

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- No load will be applied on the sides of the ladder
    - The user will only apply their weight to the rungs of the ladder
- User is 300 pounds, which accommodates most types of students
- All components that have been modified by rounding are treated as rectangular to simplify calculations
- Weight of the ladder is negligible to the amount of weight that it can support
- All the weight of the user is applied in the center of the rung

P-diagram

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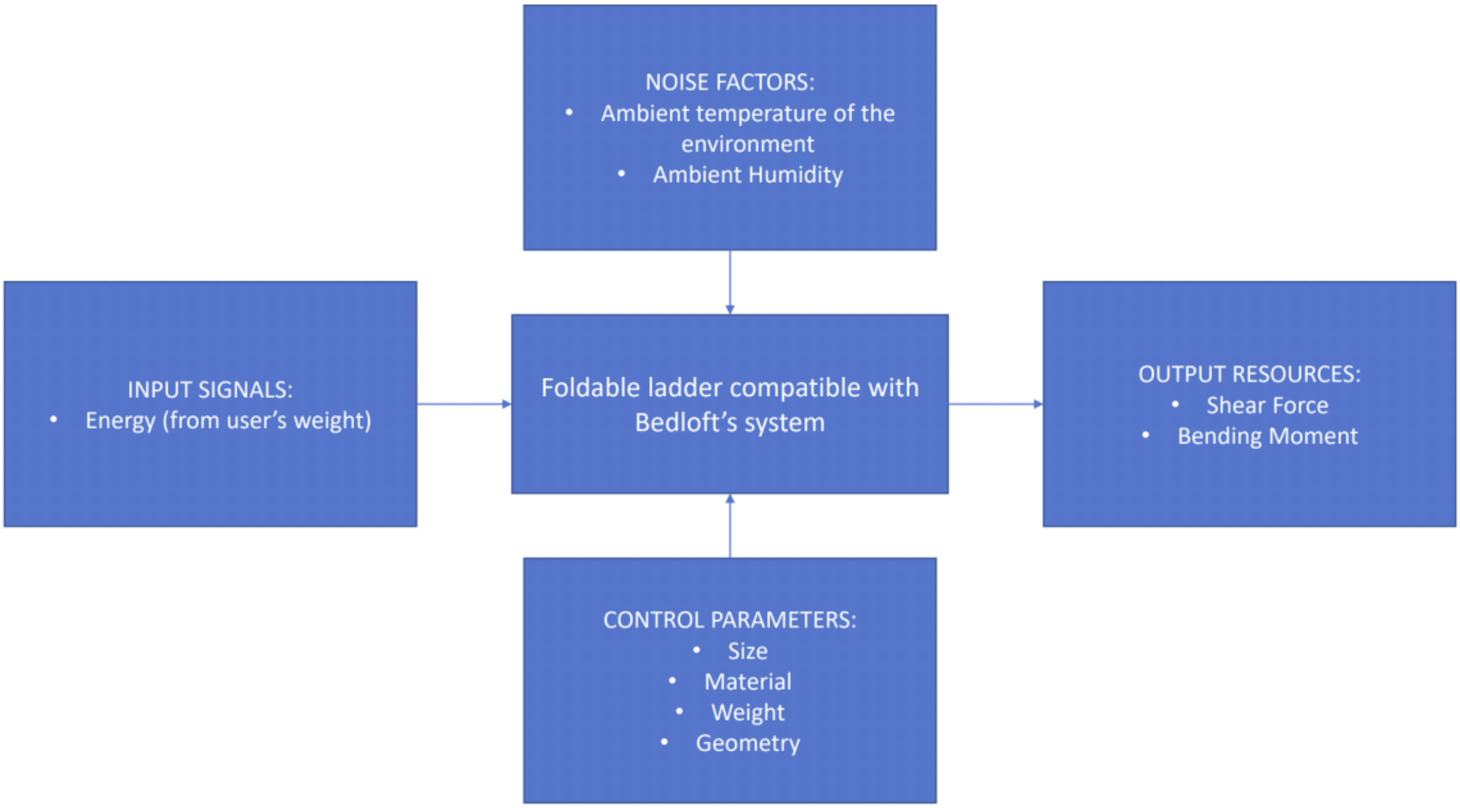

Design Variables

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→ Mass of rung (m_{rung}) = **0.67 kg**

→ Force applied by user (W_{user}) = 300 lb = **1336 N**

→ Cross-sectional area of rung (A_{rung}) = **0.0129 m^2**

→ Length of rung (l_{rung}) = **0.406 m**

→ Young's Modulus (E) = **73.1 GPa**

→ Gravitational Constant (g) = **9.8 m/s^2**

→ Thickness of rung (t_{rung}) = **0.0254 m**

→ Second Moment of Area (I) = $\frac{1}{12}(0.0508)^3(0.0254)\ m^4$ = **$2.77 \times 10^{-7}\ m^4$**

→ Stress (σ) [Pa]

→ Strain (ε) [$\frac{m}{m}$]

Simplifying Model Sketch

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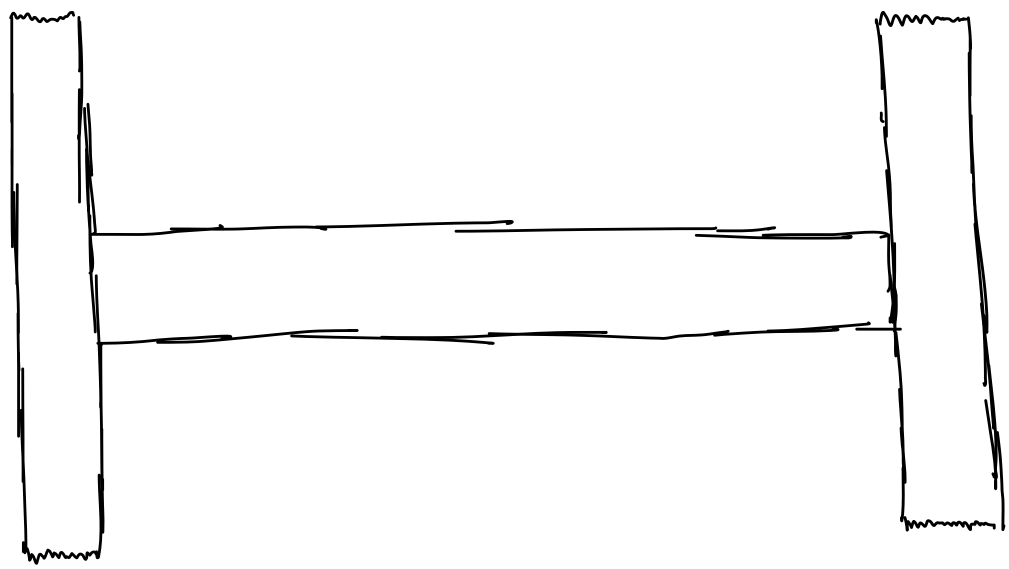

Shown above is a section of the ladder which only shows the rung and the rails. For simplifying the analysis, there is no applied load on the rails.

Free Body Diagrams

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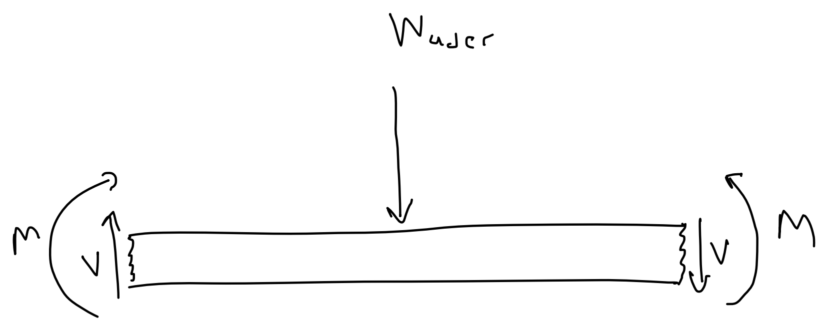
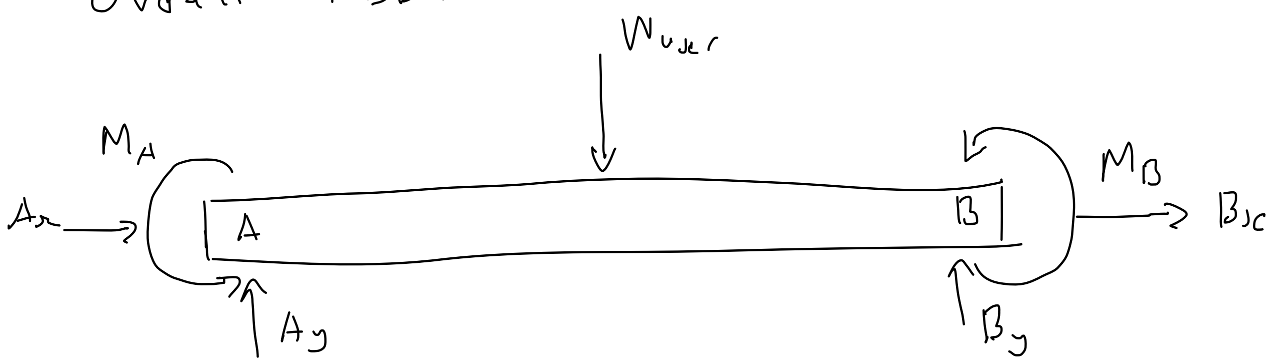
FBD of rung [internal force analysis]



Overall FBD:



Key:

- → W_{user} = weight of user
- → V = shear force
- → M = bending moment
- → A_x, A_y, M_A = reactions about A
- → B_x, B_y, M_B = reactions about B

Notes:

- * W_{user} is applied in center of rung
- * A, B are points that are attached to rails

* A and B are treated as fixed ends

Step-by-Step Derivation

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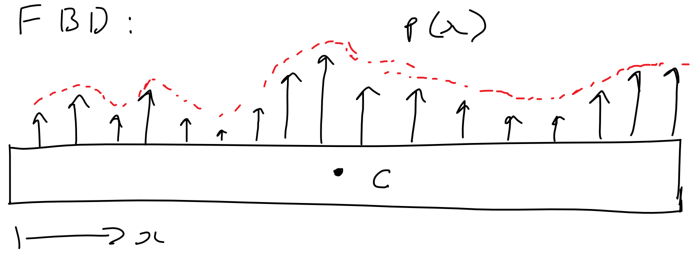
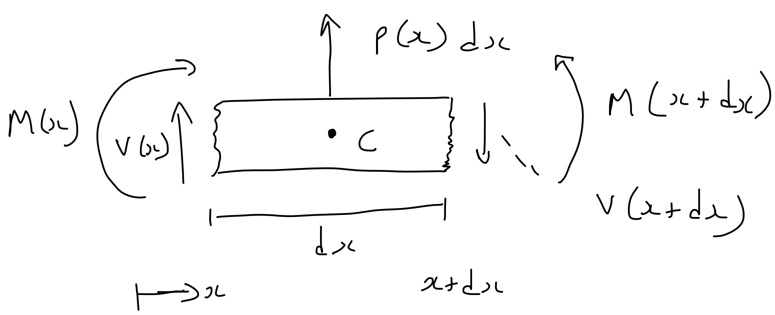
① Draw FBD:



$p(x)$ is a load described as a function.

This is a general load, and it can describe a concentrated load or one that is distributed

② Create virtual cuts:



This is a small section of the beam, also showing shear force and bending moment

③ Derivation for shear force

a) Sum forces in y-direction

$$\Sigma F_y = V(x) - V(x + dx) + p(x)\,dx = 0$$

b) Solve for $p(x)$

$$p(x)\,dx = V(x + dx) - V(x)$$

$$p(x) = \frac{V(x + dx) - V(x)}{dx}$$

c) Apply definition of derivative

$$p(x) = \frac{dV}{dx}$$

d) Solve for V

$$p(x) = \frac{dV}{dx}$$

$$dV = p(x)\,dx$$

$$\int dV = \int p(x)\,dx$$

$$\boxed{V(x) = V(0) + \int_0^x p(x)\,dx}$$

This is the shear force with respect to a distance x from a certain reference point

④ Derivation of Bending moment

a) Sum the moments around point C

$$\sum M_c = -M(x) + M(x+dx) - V(x)\frac{dx}{2} - V(x+dx)\frac{dx}{2} = 0$$

b) Solve for $V(x)$

[some steps are ommitted]

$$V(x) = \frac{M(x+dx) - M(x)}{dx}$$

c) Apply definition of derivative

$$V(x) = \frac{dM}{dx}$$

d) Solve for M

$$V(x) = \frac{dM}{dx}$$

$$dM = V(x)\,dx$$

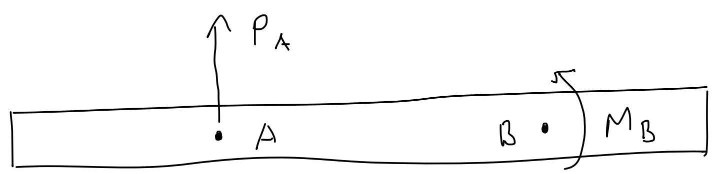
$$\int dM = \int V(x)\,dx$$

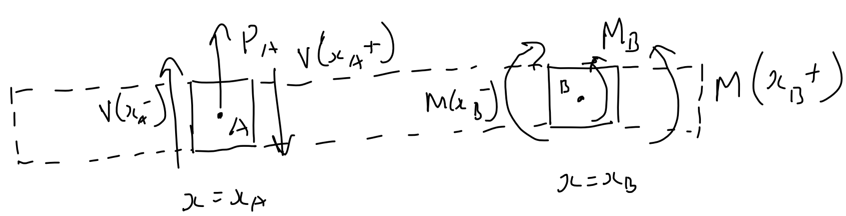
$$M(x) = M(0) + \int^x V(x)\,dx$$

$$M(x) = M(0) + \int_0^x V(x)\,dx$$

This is the bending moment with respect to a distance x from a certain reference point

(5) Concentrated loads / moments can cause discontinuity and there are times where the derivative would not exist. The limit can exist, but not the point.





At A:

(1) Sum the forces in the y-direction

$$\Sigma F_y = P_A + V(x_A^-) - V(x_A^+) = 0$$

(2) Solve for $V(x_A^+)$

$$\Rightarrow V(x_A^+) = V(x_A^-) + P_A$$

At B:

(1) Sum the moment around B

$$\Sigma M_B = M_B + M(x_B^-) - M(x_B^+) = 0$$

(2) Solve for $M(x_B^+)$

(2) Solve for $M(x_B^+)$

$$\Rightarrow \boxed{M(x_B^+) = M(x_B^-) + M_B}$$

Final Equations

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$$V(x) = V(0) + \int_0^x p(x)\,dx$$

$$M(x) = M(0) + \int_0^x V(x)\,dx$$

$$V(x_A^+) = V(x_A^-) + P_A$$

$$M(x_B^+) = M(x_B^-) + M_A$$

Engineering Units Equation Validation Check

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$$V(x) = V(0) + \int_0^x p(x)\,dx$$

$$[N] = [N] + [N] \quad \checkmark$$

$$M(x) = M(0) + \int_0^x V(x)\,dx$$

$$[N \cdot m] = [N \cdot m] + [N \cdot m] \quad \checkmark$$

$$V(x_A^+) = V(x_A^-) + P_A$$

$$[N] = [N] + [N] \quad \checkmark$$

$$M(x_B^+) = M(x_B^-) + M_A$$

$$[N \cdot m] = [N \cdot m] + [N \cdot m] \checkmark$$

Picture of Code Developed

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```
clear
clc
%% Inputs %%
Length = 0.406;
Elastic_Modulus = 73.1;
Second_Moment_of_Inertia = 2.77*10^(-7);
Weight_of_Person = 1336;
Reaction_Force_at_Ends = Weight_of_Person/2;
Reaction_Moment_at_Ends = ((Length/2)*Reaction_Force_at_Ends)/2;
x1 = linspace(0,Length/2);
x2 = linspace(Length/2,Length);

%% Calculations for 0 < x < Length/2 %%
% Shear Force Function
V1 = Reaction_Force_at_Ends;

% Bending Moment Function
M1 = -Reaction_Moment_at_Ends + V1.*x1;

%% Calculations for Length/2 < x < Length %%
% Shear Force Function
V2 = Reaction_Force_at_Ends - Weight_of_Person;

% Bending Moment Function
M2 = M1(1,100) + V2.*x1;
%% Graphing %%
% Shear Force Graph
V11 = linspace(V1, V1);
V22 = linspace(V2, V2);
V = [V11, V22];
Vx = [x1, x2];
subplot(2,1,1);
plot (Vx, V)
title('Shear Force Graph')
xlabel('Length along Rung (in m)')
ylabel('Shear Force (in N)')

% Bending Moment Graph
M = [M1, M2];
```

```
% Bending Moment Graph
M = [M1, M2];
Mx = [x1, x2];
subplot(2,1,2);
plot (Mx, M)
title('Bending Moment Graph')
xlabel('Length along Rung (in m)')
ylabel('Bending Moment (in N-m)')
```

Model Results Graph (in MATLAB)

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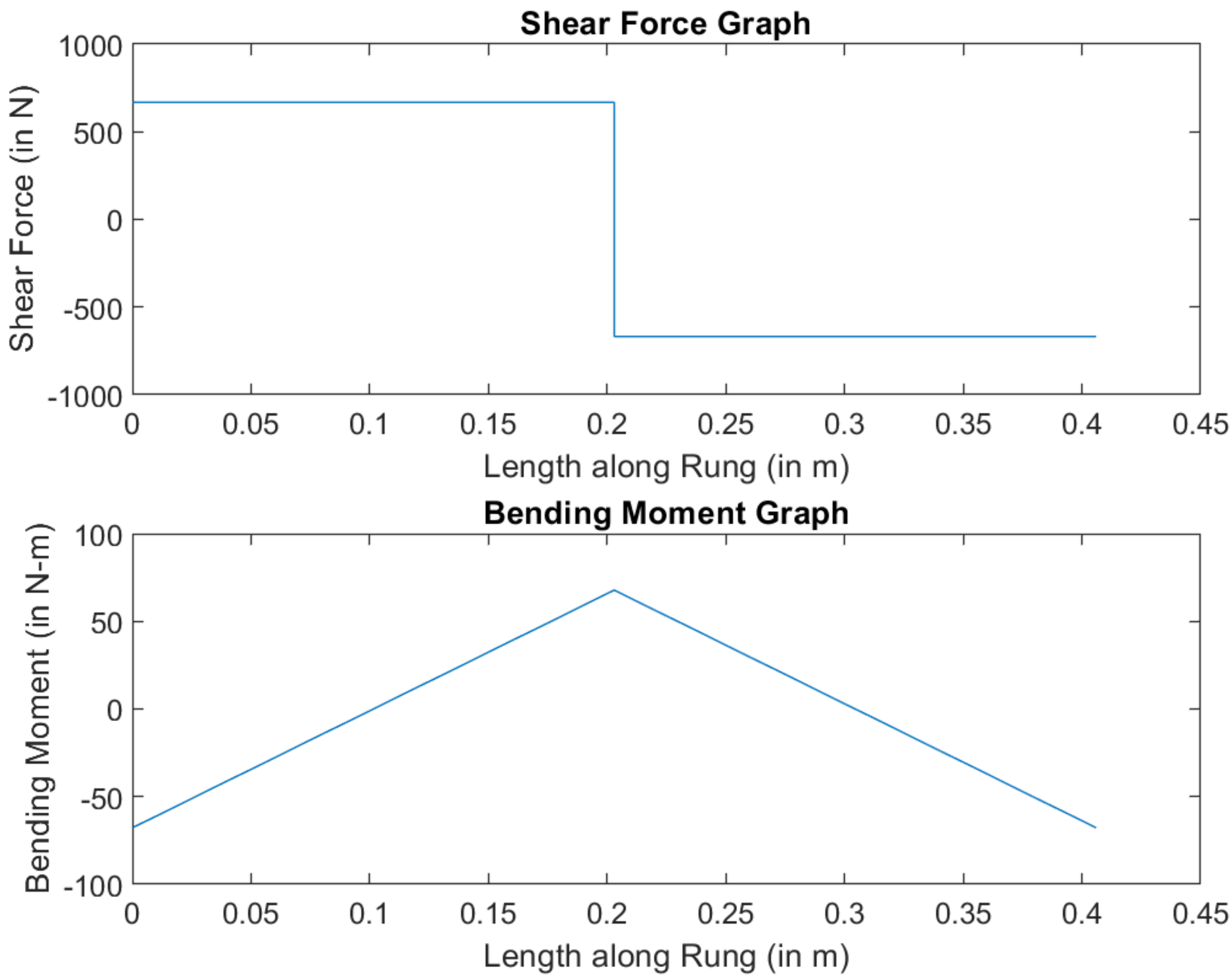

Model Results Graph (from an online calculator)

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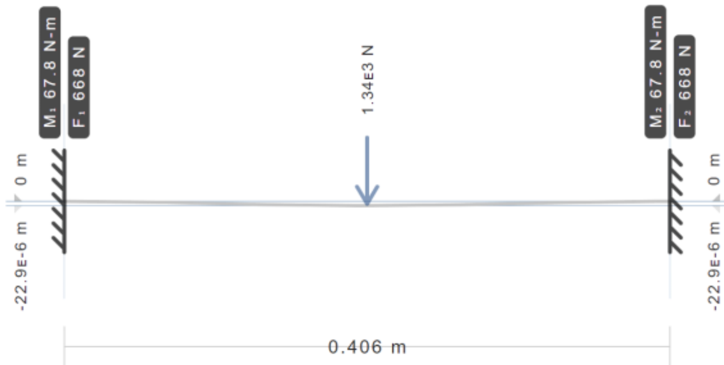

Inertia

2.77488e-7 m^4

Elastic Modulus

73.1 GPa

Length

0.406 m